NUMERICAL CALCULATION OF OPPOSING JETS OF A VISCOUS INCOMPRESSIBLE LIQUID

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The collision of two equal coaxial jets is studied. A numerical solution of the exact equations of motion and of continuity is obtained on the assumption that the isothermal flow is stationary and the properties of the liquid are constant.

For the calculation of two-dimensional flows of a viscous incompressible liquid the authors in [1] constructed an implicit finite-difference system with which numerical experiments were conducted on flows in a square having openings symmetrical relative to the axes. The equations were computed for Reynolds numbers Re = 100 and 1000. Instability of the solution was found when Re = 1000; it reacted very strongly to small changes in the boundary conditions. The flow which was used as the test to determine the stability of the method of solution is of practical interest, since such jets are often encountered in engineering, such as jet stabilization of a flame in fast streams [2], combustion of solid, gaseous, or liquid fuel in furnaces with opposing burners of the shock type [3, 4], and with dehydration of solutions and thermal treatment of dispersed materials in apparatus having opposing jets [5, 6, 7, 12].

In the present report a solution is found for the problem of the collision of two coaxial round jets of a viscous incompressible liquid discharging with equal velocities from tubes of the same diameter having flanges at the ends and located very close to one another (end to end). In the mathematical formulation of the problem we use the complete Navier – Stokes equations transformed into stream and vortex functions without eliminating any of their terms (since because of the complexity of the hydrodynamics such elimination of terms is difficult to justify correctly enough). An explicit finite-difference system constructed on the basis of the results of [8] is proposed for the numerical solution of the system of equations. In the formulation of the problem and the construction of the calculation system principal attention was paid to the stability of the solution.

From the physical arrangement it is seen that the problem is axially symmetrical and its solution should be conducted in a cylindrical coordinate system. The distance measured along the axis of symmetry from the center of the gap formed by the tubes is designated as x, while the distance from the axis of the tube in the radial direction is designated as r. We limit the region of study to a rectangle symmetrically encompassing both tubes and the space between them. We assume that the velocity distribution profile is parabolic at the entrances and the exit. To reduce the problem to dimensionless form we normalize x and r in the original system of Navier – Stokes equations using a, u, v – u_0, and P – ρu_0^2. Then we obtain the well-known equations of motion

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=-\frac{\partial P}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right), \quad (1)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}=-\frac{\partial P}{\partial r}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right), \quad (2)$$

where

$$\mathrm{Re}=u_0 a/\nu \quad (3)$$

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is the Reynolds number, and the equation of continuity

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0. \tag{4}$$

We assume that x varies in the region $-x_0 \le x \le +x_0$, while the gap occupies the subregion $-x_1 \le x \le +x_1$. Then one can write the following boundary conditions:

a) at the entrance

$$u(-x_0;\ r)=(1-r^2), \tag{5}$$

$$\frac{\partial v(-x_0;\ r)}{\partial x}=0 \tag{6}$$

and

$$u(+x_0;\ r)=-u(-x_0;\ r) \tag{5'}$$

$$\frac{\partial v(+x_0;\ r)}{\partial x}=0; \tag{6'}$$

b) at the exit

$$v(x;\ 1)=\frac{v_0}{u_0}(1-(x/x_1)^2), \tag{7}$$

$$\frac{\partial u(x;\ 1)}{\partial r}=0 \tag{8}$$

for $-x_1 \le x \le +x_1$, where v_0 is determined from the law of conservation of mass of the liquid

$$\int_0^1 u(x_0;\ r)r\,dr=\int_0^{x_1} v(x;\ 1)\,dx. \tag{9}$$

Since the exact profile of the boundary conditions at the exit is unknown, several variants of it were examined. In particular, at the cut of the gap we examined the profiles (7) and $v(x;\ 1) = v_0/u_0$ and at a certain distance away from the cut of the gap along the flanges the profile

$$v(x;\ 2)=v_0/u_0(1-(x/x_1)^2).$$

The numerical experiments showed that in all the cases compared the structure of the flow in the scale of the region studied varies little, with the streamlines shifting slightly to one side or the other from those corresponding to Fig. 2. The fact that the width of the gap is small compared with the diameter of the dispersion tubes obviously has an effect. However, the conditions of (7) are the most acceptable from considerations of stability of the problem and the feasibility of a reasonable simplification within the limits of the accuracy of the description, and are conformed by the experimental data [13] obtained by El'perin;

c) at the wall

$$u(x;\ 1)=0,\quad v(x;\ 1)=0 \tag{10}$$

for $-x_0 \le x \le -x_1$ and $+x_1 \le x \le +x_0$.

The assigning of the pressure distribution at the boundary is complicated in internal problems and therefore we will change from the system of equations (1)-(4) to equations in vortex and stream functions, which do not contain the pressure. According to (4), a stream function $\psi(x;\ r)$ exists for which

$$u=\frac{1}{r}\frac{\partial\psi}{\partial r},\qquad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}. \tag{11}$$

Determining the vortex velocity ω' by the equation

$$\omega'=\frac{1}{r}\left(\frac{\partial u}{\partial r}-\frac{\partial v}{\partial x}\right), \tag{12}$$

we obtain

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial r^2}=r^2\omega'+\frac{1}{r}\frac{\partial\psi}{\partial r}. \tag{13}$$

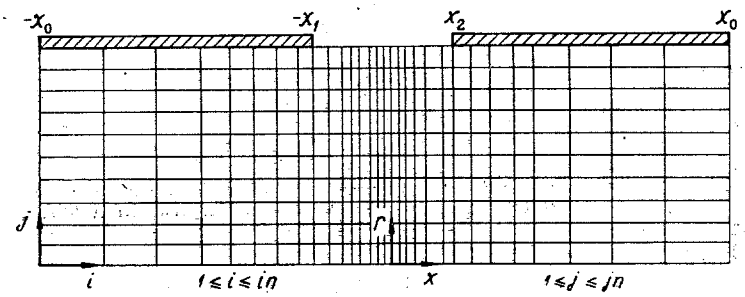

Fig. 1. Diagram showing the region of integration, the grid, and the variables used in the problem.

Differentiating (1) and (2) with respect to x and r, respectively, and subtracting one equation from the other, we obtain

$$\frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial x}\left(r^3\frac{\partial\omega'}{\partial x}\right)+\frac{\partial}{\partial r}\left(r^3\frac{\partial\omega'}{\partial r}\right)\right]=r^2\left[\frac{\partial}{\partial x}\left(\omega'\frac{\partial\psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\omega'\frac{\partial\psi}{\partial x}\right)\right]. \tag{14}$$

Equations (13) and (14) must be solved for the unknown functions ψ and ω'.

In the subsequent discussion it is necessary to use functions in grid coordinates. Keeping in mind that under the conditions of the problem the dimensions of the gap are considerably smaller than the dimensions of the sections of the tubes under consideration, we introduce a grid which is nonuniform along the x axis, bunching together from the periphery to the center (Fig. 1). The grid remains uniform along the r axis.

Let us convert the boundary conditions from the functions u and v to the functions ψ and ω':

a) at the entrance

$$\psi(-x_0;\ r)=0.5(r^2-0.5r^4), \tag{15}$$

$$\psi(+x_0;\ r)=-\psi(-x_0;\ r). \tag{16}$$

Then it follows from Eqs. (5), (6), and (12) that

$$\omega'(-x_0;\ r)=-2, \tag{17}$$

$$\omega'(+x_0;\ r)=-\omega'(-x_0;\ r). \tag{18}$$

Similarly we obtain the following equations:

b) at the exit

$$\psi(x;\ 1)=\frac{v_0}{u_0}\,x\left((x/x_1)^2\frac{1}{3}-1\right), \tag{19}$$

$$\omega'(x;\ 1)=2\frac{v_0}{u_0}\frac{x}{x_1^2}; \tag{20}$$

c) at the wall

$$\psi(x;\ 1)=\mathrm{sign}(x)\psi(x_0;\ 1), \tag{21}$$

where

$$\mathrm{sign}(x)=\begin{cases}-1, & \text{if } x<0,\\ +1, & \text{if } x>0.\end{cases}$$

At the points $(-x_1;\ 1)$ and $(+x_1;\ 1)$ the boundary conditions undergo a discontinuity, which is due to the physical problem.

Certain complications arise in writing the boundary conditions for ω' at the wall, and ω' is not assigned directly but derived from an expansion of ψ is a Taylor series.

Let us use one of the equations obtained in [9] which preserves the stability of the problem:

$$\omega'_{i,jn}=\frac{3}{\rho r_{jn}(\Delta r_{jn})^2}(\psi_{i,jn}-\psi_{i,jn-1})-0.5\,\omega'_{i,jn-1}, \tag{22}$$

where $\Delta r_{jn} = r_{jn} - r_{jn-1}$ is the step along the r axis;

d) at the axis of symmetry

$$\psi_{i,1} = \psi_{i,2} \tag{23}$$

the equation for ω' is obtained from the condition that ω' = const at the axis of symmetry and has the form

$$\omega'_{i,1} = \omega'_{i,2} + (r_2 - r_1)/(r_3 - r_2)\ (\omega'_{i,2} - \omega'_{i,3}). \tag{24}$$

The system (13)-(14) with the boundary conditions (15)-(24) was solved numerically by the grid method. A uniform locally single-compartment difference system in which all the spatial derivatives were approximated by the central differences with an error $O(h^2)$ [10] was initially selected for the solution of the system (13)-(14). However, the numerical experiments showed that this system is unsuitable for a boundary problem in which the convection terms of the equations have large coefficients while the boundary conditions which are different from zero along the entire boundary vary strongly, since stability of the solution was achieved only for small Reynolds numbers Re = 10. Henceforth we used for the calculations a conservative uniform finite-difference system constructed by the integro-interpolation method and based on ideas developed in [8-11]. So that the difference system would provide the possibility of obtaining a solution for high Reynolds numbers, in its construction the convection terms were approximated by one-sided differences instead of central differences, with the direction of motion of the liquid being taken into account to improve the conservativity of the system.

In the difference form the system (13)-(14) is written in the form

$$\omega'_{i,j} = (A_1\omega'_{i+1,j} + A_2\omega'_{i-1,j} + A_3\omega'_{i,j+1} + A_4\omega'_{i,j-1}) \Big/ \sum_{k=1}^{4} A_k, \tag{25}$$

$$\psi_{i,j} = (B_1\psi_{i+1,j} + B_2\psi_{i-1,j} + B_3\psi_{i,j+1} + B_4\psi_{i,j-1}) \Big/ \sum_{k=1}^{4} B_k, \tag{26}$$

where

$$A_1 = 0.5 r_j^2 \left[(\psi_{i+\frac{1}{2},\, j-\frac{1}{2}} - \psi_{i+\frac{1}{2},\, j+\frac{1}{2}}) + |\psi_{i+\frac{1}{2},\, j-\frac{1}{2}} - \psi_{i+\frac{1}{2},\, j+\frac{1}{2}}| \right] + \frac{r_j^3 (r_{j+1} - r_{j-1})}{4\mathrm{Re}\,(x_{i+1} - x_i)},$$

$$A_2 = 0.5 r_j^2 \left[(\psi_{i-\frac{1}{2},\, j+\frac{1}{2}} - \psi_{i-\frac{1}{2},\, j-\frac{1}{2}}) + |\psi_{i-\frac{1}{2},\, j+\frac{1}{2}} - \psi_{i-\frac{1}{2},\, j-\frac{1}{2}}| \right] + \frac{r_j^3 (r_{j+1} - r_{j-1})}{4\mathrm{Re}\,(x_i - x_{i-1})},$$

$$A_3 = 0.5 r_j^2 \left[(\psi_{i+\frac{1}{2},\, j+\frac{1}{2}} - \psi_{i-\frac{1}{2},\, j+\frac{1}{2}}) + |\psi_{i+\frac{1}{2},\, j+\frac{1}{2}} - \psi_{i-\frac{1}{2},\, j+\frac{1}{2}}| \right] + \frac{(r_{j+1} + r_j)^3 (x_{i+1} - x_{i-1})}{8\,\mathrm{Re}\,(r_{j+1} + r_j)},$$

$$A_4 = 0.5 r_j^2 \left[(\psi_{i-\frac{1}{2},\, j-\frac{1}{2}} - \psi_{i+\frac{1}{2},\, j-\frac{1}{2}}) + |\psi_{i-\frac{1}{2},\, j-\frac{1}{2}} - \psi_{i+\frac{1}{2},\, j-\frac{1}{2}}| \right] + \frac{(r_{j-1} + r_j)^3 (x_{i+1} - x_{i-1})}{8\,\mathrm{Re}\,(r_j - r_{j-1})},$$

$$B_1 = \frac{1}{4r_j} \frac{r_{j+1} - r_{j-1}}{x_{i+1} - x_i}, \quad B_2 = \frac{1}{4r_j} \frac{r_{j+1} - r_{j-1}}{x_i - x_{i-1}},$$

$$B_3 = \frac{r_{j+1}^{-1} + r_j^{-1}}{8} \frac{x_{i+1} - x_{i-1}}{r_{j+1} - r_j}, \quad B_4 = \frac{r_{j-1}^{-1} + r_j^{-1}}{8} \frac{x_{i+1} - x_{i-1}}{r_j - r_{j-1}},$$

$$B_5 = \omega'_{i,j} r_j \left(\frac{x_{i+1} - x_{i-1}}{2} \right) \left(\frac{r_{j+1} - r_{j-1}}{2} \right).$$

Here it was assumed with respect to the stream function ψ that its value at intermediate points is equal to the arithmetic mean of the values at the four adjacent nodes, for example,

$$\psi_{i+\frac{1}{2},\, j+\frac{1}{2}} \approx \frac{1}{4} (\psi_{i,j+1} + \psi_{i,j} + \psi_{i+1,j} + \psi_{i+1,j+1}),$$

where $i \pm 1/2$, $j \pm 1/2$ are intermediate points of the grid.

The system of difference equations (25)-(26) was solved by the Seidel iteration method. As the first approximation we took either $\psi_{i,j} = \omega'_{i,j} = -0.01$ sign (x) or the solution obtained with any other value of the parameter of the process. After each iteration step the boundary values were calculated for the vorticity and for the stream function at the axis of symmetry. Having obtained the solution of Eqs. (13)-(14), we

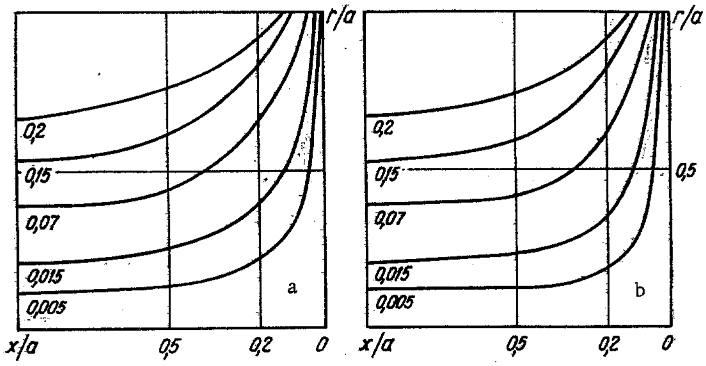

Fig. 2. Streamlines for Re = 10 (a) and Re = 10^6 (b).

determine the distribution of the velocities u and v from Eqs. (11) and we obtain $\partial P/\partial x$ or $\partial P/\partial r$ from (1)–(2) and the pressure from the recurrent equation

$$P_b \approx P_a + 0.5\left[\left(\frac{\partial P}{\partial x}\right)_a + \left(\frac{\partial P}{\partial x}\right)_b\right](x_b - x_a)$$

The calculations were conducted on a 43 × 20 grid which was nonuniform along the x axis, 23 of the nodes being assigned in the subregion of the gap $-x_1 \le x \le +x_1$. In the assignment of the nodes we were constrained to observe the condition that the ratio of sizes of adjacent intervals did not exceed about 1.5 or else distortion of the pattern was observed and on the curve, which should be smooth, bends appeared which were the larger, the more strongly this condition was violated. It did not seem possible to obtain results on a uniform grid since the width of the gap would become comparable with the step of the grid. A comparison of the results on 43 × 20 grids with different distributions of the nodes and with control results on a 63 × 30 grid showed that the discrepancies are insignificant. In the numerical experiments it was considered that the iteration process had converged when the maximum relative change in the variables between successive iterations was less than 0.005, and this took 200–400 iterations or 2–4 h depending on the operating parameters of the Minsk-32 electronic computer in the mode of compatibility.

In the problem Re and x_1 were analyzed and the following experiments were set up: with x_1 = const = 0.2 the Reynolds number was varied in the range of $1 \le \mathrm{Re} \le 10^6$, and with Re = const = 66,000 we varied x_1 in the range of $0.2 \ge x_1 \ge 0.08$.

Streamline patterns are presented in Fig. 2 for $x_1 = 0.2$ and Re = 10 (a) and Re = 10^6 (b).

The flow structure obtained theoretically agrees qualitatively with the general flow structure which can be expected on the basis of experimental data. As is seen from the figures, the motion in the chamber has a layered nature and the presence of vortices was not observed. With an increase in the Reynolds number the streamlines are slightly deformed in the direction of the surface of contact between the jets.

In the collision of two jets of equal diameter having the same velocities the contact surface consists of a plane located perpendicular to the axes of the jets.

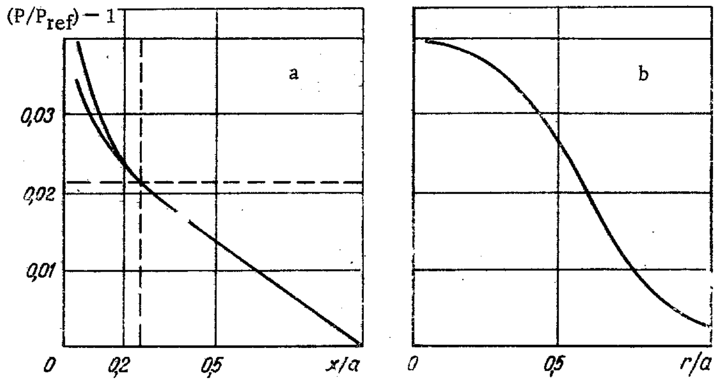

Fig. 3. Pressure distribution along chamber axis (a) and at the surface of collision of the jets (b) in the form $(P/P_{ref}) - 1$.

The pressure distribution over the chamber is interesting, especially in the region of mutual contact of the jets, and is characterized by considerable pressure gradients both in the direction of the chamber axis (Fig. 3a) and in the perpendicular plane when $x_1 = 0.2$ (Fig. 3b). Graphs of the pressure distribution along the chamber for the parameters $x_1 = 0.2$ and 0.08 and Re = 66,000 are shown in Fig. 3a.

The numerical experiments showed that the pressure variation in the tubes does not depend on the distance between them. Bringing the ends of the pipelines closer together leads to an increase in the pressure only in the zone of contact between the jets, the border of which is comparable with the width of the gap-outlet. The pressure increases linearly in the tubes upon approach to the contact surface, it increases more sharply in the zone of contact between the jets, and finally reaches its maximum value at the critical point.

In conclusion, it should be noted that the method of solution used showed satisfactory stability. However, the results obtained for large Reynolds numbers must be approached with caution since a fictitious diffusion, which somewhat distorts the exact solution, shows up as a general defect of methods which use one-sided differences.

NOTATION

ρ is the density of liquid;
ν is the kinematic viscosity;
μ is the dynamic viscosity coefficient;
a is the radius of tube;
u is the velocity in axial direction;
v is the velocity in radial direction;
P is the hydrodynamic pressure;
u_0 is the initial velocity;
l is the length of chamber.

LITERATURE CITED

1. K. B. Dzhakulov and B. G. Kuznetsov, Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 1, 95-99 (1969).
2. L. A. Vulis and V. P. Kashkarov, Theory of Jets of Viscous Liquid [in Russian], Nauka, Moscow (1965).
3. Yu. P. Enyakin and A. I. Dvoretskii, Teploenergetika, No. 2 (1968).
4. Yu. P. Enyakin, A. D. Gorbanenko, A. M. Dvoretskii, and L. M. Tsiryul'nikov, Energet. Stantsii, No. 7 (1968).
5. L. L. Pavlovskii, B. K. Tel'nov et al., New Equipment and Instruments for the Production and Evaluation of Construction Materials [in Russian], No. 23 (31), Moscow (1969).
6. Galershtein et al., in: Study of Heat and Mass Exchange in Technological Processes and Apparatus [in Russian], Nauka i Tekhnika, Minsk (1966).
7. V. L. Mel'tser and I. T. Él'perin, in: Study of Transport Processes in Apparatus Containing Dispersed Systems [in Russian], Nauka i Tekhnika, Minsk (1969).
8. H. L. Barakat and I. A. Clark, Proceedings of Third International Heat-Transfer Conference, Chicago, 1966, Vol. 2, No. 1, Amer. Inst. Chem. Engrs. (1966), pp. 152-162.
9. T. V. Kuskova and L. A. Chudov, Izv. Sibirsk. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 8, Pt. 2, 92-96 (1967).
10. A. A. Samarskii, Zh. Vychisl. Matem. i Matem. Fiz., 2, No. 5, 787-811 (1962).
11. A. A. Samarskii, Introduction to the Theory of Difference Systems [in Russian], Nauka, Moscow (1971).
12. P. S. Kuts, É. G. Tutova, and G. S. Kabaldin, Inzh.-Fiz. Zh., 24, No. 4 (1973).
13. I. T. Él'perin, Transport Processes in Opposing Jets (Gas Suspension) [in Russian], Nauka i Tekhnika, Minsk (1972).